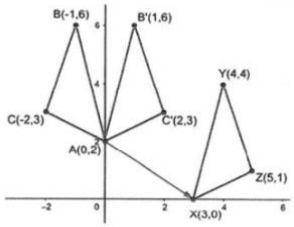
A Side-by-Side Comparison of the Geometry with Data Analysis Standards
in the
2019 Alabama Course of Study: Mathematics and the Common Core State Standards for Mathematics

	2019 Alabama Course of Study: Mathematics		Common Core State Standards for Mathematics
1	Extend understanding of irrational and rational numbers by rewriting expressions involving radicals, including addition, subtraction, multiplication, and division, in order to recognize geometric patterns.		
2	Use units as a way to understand problems and to guide the solution of multi-step problems. a. Choose and interpret units consistently in formulas. b. Choose and interpret the scale and the origin in graphs and data displays. c. Define appropriate quantities for the purpose of descriptive modeling. d. Choose a level of accuracy appropriate to limitations of measurements when reporting quantities.	N-Q.1 N-Q.2 N-Q.3	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. Define appropriate quantities for the purpose of descriptive modeling. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.
3	Find the coordinates of the vertices of a polygon determined by a set of lines, given their equations, by setting their function rules equal and solving, or by using their graphs.		
4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. Example: Rearrange the formula for the area of a trapezoid to highlight one of the bases.	A-CED.4	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law V = IR to highlight resistance R. ★
5	Verify that the graph of a linear equation in two variables is the set of all its solutions plotted in the coordinate plane, which forms a line.		
6	Derive the equation of a circle of given center and radius using the Pythagorean Theorem. a. Given the endpoints of the diameter of a circle, use the midpoint formula to find its center and then use the Pythagorean Theorem to find its equation. b. Derive the distance formula from the Pythagorean Theorem.	G-GPE.1	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
7	Use mathematical and statistical reasoning with quantitative data, both univariate data (set of values) and bivariate data (set of pairs of values) that suggest a linear association, in order to draw conclusions and assess risks. Example: Estimate the typical age at which a lung cancer patient is diagnosed, and estimate how the typical age differs depending on the number of cigarettes smoked per day.		
8	Use technology to organize data, including very large data sets, into a useful and manageable structure.		

9	Represent the distribution of univariate quantitative data with plots on the real number line, choosing a format (dot plot, histogram, or box plot) most appropriate to the data set, and represent the distribution of bivariate quantitative data with a scatter plot. Extend from simple cases by hand to more complex cases involving large data sets using technology.	S-ID.1	Represent data with plots on the real number line (dot plots, histograms, and box plots). ★
10	Use statistics appropriate to the shape of the data distribution to compare and contrast two or more data sets, utilizing the mean and median for center and the interquartile range and standard deviation for variability.	S-ID.2	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. ★
11	Interpret differences in shape, center, and spread in the context of data sets, accounting for possible effects of extreme data points (outliers) on mean and standard deviation.	S-ID.3	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★
12	Represent data of two quantitative variables on a scatter plot, and describe how the variables are related. a. Find a linear function for a scatter plot that suggests a linear association and informally assess its fit by plotting and analyzing residuals, including the squares of the residuals, in order to improve its fit. b. Use technology to find the least-squares line of best fit for two quantitative variables.	S-ID.6	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. ★ a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.
13	Compute (using technology) and interpret the correlation coefficient of a linear relationship.	S-ID.8	Compute (using technology) and interpret the correlation coefficient of a linear fit. ★
14	Distinguish between correlation and causation.	S-ID.9	Distinguish between correlation and causation. ★
15	Evaluate possible solutions to real-life problems by developing linear models of contextual situations, predicting unknown values. a. Use the linear model to solve problems in the context of the given data. b. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the given data.	S-ID.7	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. ★
16	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	G-GMD.4	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
17	Model and solve problems using surface area and volume of solids, solids and solids with portions removed. a. Give an informal argument for the formulas for the surface area and volume of a sphere, cylinder, pyramid, and cone using dissection arguments, Cavalieri's Principle, and informal limit arguments. b. Apply geometric concepts to find missing dimensions to solve surface area or volume problems.	G-GMD.1	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
		G-GMD.2	(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
		G-GMD.3	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★

18	Given the coordinates of the vertices of a polygon, compute its perimeter and area using a variety of methods, including the distance formula and dynamic geometry software, and evaluate the accuracy of the results.	G-GPE.7	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★
19	Derive and apply the relationships between the lengths, perimeters, areas, and volumes of similar figures in relation to their scale factor.		
20	Derive and apply the formula for the length of an arc and the formula for the area of a sector.	G-C.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
21	Represent transformations and compositions of transformations in the plane (coordinate and otherwise) using tools such as tracing paper and geometry software. a. Describe transformations and compositions of transformations as functions that take points in the plane as inputs and give other points as outputs, using informal and formal notation. b. Compare transformations which preserve distance and angle to those that do not.	G-CO.2	Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
22	Explore rotations, reflections, and translations using graph paper, tracing paper, and geometry software. a. Given a geometric figure and a rotation, reflection, or translation, draw the image of the transformed figure using graph paper, tracing paper, or geometry software. b. Specify a sequence of rotations, reflections, or translations that will carry a given figure onto another. c. Draw figures with different types of symmetries and describe their attributes.	G-CO.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
23	Develop definitions of rotation, reflection, and translation in terms of angles, circles, perpendicular lines, parallel lines, and line segments.	G-CO.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
24	Define congruence of two figures in terms of rigid motions (a sequence of translations, rotations, and reflections); show that two figures are congruent by finding a sequence of rigid motions that maps one figure to the other. Example: ΔABC is congruent to ΔXYZ Since a reflection followed by a translation maps ΔABC onto ΔXYZ.	G-CO.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

25	 Verify criteria for showing triangles are congruent using a sequence of rigid motions that map one triangle to another. a. Verify that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. b. Verify that two triangles are congruent if (but not only if) the following groups of corresponding parts are congruent: angle-side-angle (ASA), side-angle-side (SAS), side-side-side (SSS), and angle-angle-side (AAS). Example: Given two triangles with two pairs of congruent corresponding sides and a pair of congruent included angles, show that a sequence of rigid motions will map one onto the other.	G-CO.7 G-CO.8	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
26	Verify experimentally the properties of dilations given by a center and a scale factor. a. Verify that a dilation takes a line not passing through the center of the dilation to a a parallel line, and leaves a line passing through the center unchanged. b. Verify that the dilation of a line segment is longer or shorter in the ratio given by the scale factor.	G-SRT.1	Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
27	Given two figures, determine whether they are similar by identifying a similarity transformation that maps one figure to the other.	G-SRT.2	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
28	28. Verify criteria for showing triangles are similar using a similarity transformation (sequence of rigid motions and dilations) that maps one triangle to another. a. Verify that two triangles are similar if and only if corresponding pairs of sides are proportional and corresponding pairs of angles are congruent. b. Verify that two triangles are similar if (but not only if) two pairs of corresponding angles are congruent (AA), the corresponding sides are proportional (SSS), or two pairs of corresponding sides are proportional and the pair of included angles is congruent (SAS). Example: Given two triangles with two pairs of congruent corresponding sides and a pair of congruent included angles, show there must be a set of rigid motions that maps one onto the other.	G-SRT.3	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

29	Find patterns and relationships in figures including lines, triangles, quadrilaterals, and circles, using technology and other tools. a. Construct figures, using technology and other tools, in order to make and test conjectures about their properties. b. Identify different sets of properties necessary to define and construct figures.	G-CO.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
30	Develop and use precise definitions of figures such as angle, circle, perpendicular lines, parallel lines, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	G-CO.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
31	Justify whether conjectures are true or false in order to prove theorems and then apply those theorems in solving problems, communicating proofs in a variety of ways, including flow chart, two-column, and paragraph formats. a. Investigate, prove, and apply theorems about lines and angles, including but not limited to: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; the points on the perpendicular bisector of a line segment are those equidistant from the segment's endpoints.	G-CO.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.
	b. Investigate, prove, and apply theorems about triangles, including but not limited to: the sum of the measures of the interior angles of a triangle is 180°; the base angles of isosceles triangles are congruent; the segment joining the midpoints of two sides of a triangle is parallel to the third side and half the length; a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem can be proved using triangle similarity.	G-CO.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.
		G-SRT.4	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
	c. Investigate, prove, and apply theorems about parallelograms and other quadrilaterals, including but not limited to both necessary and sufficient conditions for parallelograms and other quadrilaterals, as well as relationships among kinds of quadrilaterals. Example: Prove that rectangles are parallelograms with congruent diagonals.	G-CO.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals.
32	Use coordinates to prove simple geometric theorems algebraically.		
33	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems. Example: Find the equation of a line parallel or perpendicular to a given line that passes through a given point.	G-GPE.5	Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
34	Use congruence and similarity criteria for triangles to solve problems in real-world contexts.	G-SRT.5	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

35	Discover and apply relationships in similar right triangles. a. Derive and apply the constant ratios of the sides in special right triangles (45°-45°-90°) and 30°-60°-90°). b. Use similarity to explore and define basic trigonometric ratios, including sine ratio, cosine ratio, and tangent ratio. c. Explain and use the relationship between the sine and cosine of complementary angles. d. Demonstrate the converse of the Pythagorean Theorem. e. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems, including finding areas of regular polygons.	G-SRT.6	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
		G-SRT.7	Explain and use the relationship between the sine and cosine of complementary angles.
		G-SRT.8	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★
		G-SRT.10	(+) Prove the Laws of Sines and Cosines and use them to solve problems.
		G-SRT.11	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
36	Use geometric shapes, their measures, and their properties to model objects and use those models to solve problems.	G-MG.1	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★
37	Investigate and apply relationships among inscribed angles, radii, and chords, including but not limited to: the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	G-C.2	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
38	Use the mathematical modeling cycle involving geometric methods to solve design problems. Examples: Design an object or structure to satisfy physical constraints or minimize cost; work with typographic grid systems based on ratios; apply concepts of density based on area volume.	G-MG.3	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★

A Side-by-Side Comparison of the Algebra I with Probability Standards
in the
2019 Alabama Course of Study: Mathematics and the Common Core State Standards for Mathematics

	2019 Alabama Course of Study: Mathematics		Common Core State Standards for Mathematics
1	Explain how the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for an additional notation for radicals using rational exponents.	N-RN.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*
2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	N-RN.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
3	Define the imaginary number *i* such that $i^2 = -1$.	N-CN.1	Know there is a complex number i such that $i^2 = –1$, and every complex number has the form a + bi with a and b real.
4	Interpret linear, quadratic, and exponential expressions in terms of a context by viewing one or more of their parts as a single entity. Example: Interpret the accrued amount of investment $P(1 + r)^t$, where P is the principal and r is the interest rate, as the product of P and a factor depending on time t.	A-SSE.1	Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.
5	Use the structure of an expression to identify ways to rewrite it. Example: See $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.	A-SSE.2	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
6	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor quadratic expressions with leading coefficients of one, and use the factored form to reveal the zeros of the function it defines. b. Use the vertex form of a quadratic expression to reveal the maximum or minimum value and the axis of symmetry of the function it defines; complete the square to find the vertex form of quadratics with a leading coefficient of one. c. Use the properties of exponents to transform expressions for exponential functions. Example: Identify percent rate of change in functions such as $y = (1 .02)^t$, $y = (0.97)^t$, $y = (1 .01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.	A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
7	Add, subtract, and multiply polynomials, showing that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication.	A-APR.1	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

8	Explain why extraneous solutions to an equation involving absolute values may arise how to check to be sure that a candidate solution satisfies an equation.	A-REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
9	Select an appropriate method to solve a quadratic equation in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x — p)^2 = q$ that has the same solutions. Explain how the solution is derived from this form. b. Solve quadratic equations by inspection (such as $x^2 = 49$), taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation, and recognize that some solutions may not be real.	A-REI.4	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
10	Select an appropriate method to solve a system of two linear equations in two variables. a. Solve a system of two equations in two variables by using linear combinations; contrast situations in which use of linear combinations is more efficient with those in which substitution is more efficient. b. Contrast solutions to a system of two linear equations in two variables produced by algebraic methods with graphical and tabular methods.	A-REI.6	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
		A-REI.5	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
11	Create equations and inequalities in one variable and use them to solve problems in context, either exactly or approximately. Extend from contexts arising from linear functions to those involving quadratic, exponential, and absolute value functions.	A-CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. ★
12	Create equations in two or more variables to represent relationships between quantities in context; graph equations on coordinate axes with labels and scales and use them to make predictions. Limit to contexts arising from linear, quadratic, exponential, absolute value, and general piecewise functions.	A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
13	Represent constraints by equations and/or inequalities, and solve systems of equations and/or inequalities, interpreting solutions as viable or nonviable options in a modeling context. Limit to contexts arising from linear, quadratic, exponential, absolute value, and general piecewise functions.	A-CED.3	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non- viable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods. ★
14	Given a relation defined by an equation in two variables, identify the graph of the relation as the set of all its solutions plotted in the coordinate plane. Note: The graph of a relation often forms a curve (which could be a line).	A-REI.10	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
15	Define a function as a mapping from one set (called the domain) to another set (called the range) that assigns to each element of the domain exactly one element of the range. a. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms	F-IF.1	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

<table>
<tr><td rowspan="2"></td><td rowspan="2">of a context. Note: If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x.
b. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Limit to linear, quadratic, exponential, and absolute value functions.</td><td>F-IF.2</td><td>Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.</td></tr>
<tr><td>F-IF.5</td><td>Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★</td></tr>
<tr><td>16</td><td>Compare and contrast relations and functions represented by equations, graphs, or tables that show related values; determine whether a relation is a function. Identify that a function f is a special kind of relation defined by the equation y =f(x).</td><td></td><td></td></tr>
<tr><td>17</td><td>Combine different types of standard functions to write, evaluate, and interpret functions in context. Limit to linear, quadratic, exponential, and absolute value functions.
a. Use arithmetic operations to combine different types of standard functions to write and evaluate functions. Example: Given two functions, one representing flow rate of water and the other representing evaporation of that water, combine the two functions to determine the amount of water in a container at a given time.
b. Use function composition to combine different types of standard functions to write and evaluate functions. Example: Given the following relationships, determine what the expression S(T(t)) represents.
<table><tr><td>Function</td><td>Input</td><td>Output</td></tr><tr><td>G</td><td>Amount of studying: s</td><td>Grade in course: G(s)</td></tr><tr><td>S</td><td>Grade in course: g</td><td>Amount of screen time: S(g)</td></tr><tr><td>T</td><td>Amount of screen time: t</td><td>Number of followers: T(t)</td></tr></table></td><td>F-BF.1</td><td>Write a function that describes a relationship between two quantities.★
a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.</td></tr>
<tr><td>18</td><td>Solve systems consisting of linear and/or quadratic equations in two variables graphically, using technology where appropriate.</td><td>A-REI.7</td><td>Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line y = –3x and the circle x2 + y2 = 3.</td></tr>
<tr><td>19</td><td>Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x).
a. Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. Note: Include cases where f(x) is a linear, quadratic, exponential, or absolute value function and g(x) is constant or linear.</td><td>A-REI.11</td><td>Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★</td></tr>
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20	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes, using technology where appropriate.	A-REI.12	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
21	Compare properties of two functions, each represented in a different way (algebraically, numerically in tables, or by verbal descriptions). Extend from linear to quadratic, exponential, absolute value, and general piecewise.	F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
22	Define sequences as functions, including recursive definitions, whose domain is a subset of the integers. a. Write explicit and recursive formulas for arithmetic and geometric sequences and connect them to linear and exponential functions. Example: A sequence with constant growth will be a linear function, while a sequence with proportional growth will be an exponential function.	F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.
		F-BF.1	Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
23	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k \cdot f(x)$, $f(k \cdot x)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and explain the effects on the graph, using technology as appropriate. Limit to linear, quadratic, exponential, absolute value, and general piecewise functions.	F-BF.3	Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
24	Distinguish between situations that can be modeled with linear functions and those that can be modeled with exponential functions. a. Show that linear functions grow by equal differences over equal intervals, while exponential functions grow by equal factors over equal intervals. b. Define linear functions to represent situations in which one quantity changes at a constant rate per unit interval relative to another. c. Define exponential functions to represent situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	F-LE.1	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c, Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

25	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	F-LE.2	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
26	Use graphs and tables to show that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically.	F-LE.3	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
27	Interpret the parameters of functions in terms of a context. Extend from linear functions, written in the form mx + b, to exponential functions, written in the form ab^x. Example: If the function V(t) = $19885(0.75)^t$ describes the value of a car after it has been owned for t years, 19885 represents the purchase price of the car when t = 0, and 0.75 represents the annual rate at which its value decreases.	F-LE.5	Interpret the parameters in a linear or exponential function in terms of a context.
28	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Note: Key features include; intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries; and end behavior. Extend from relationships that can be represented by linear functions to quadratic, exponential, absolute value, and general piecewise functions.	F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★
29	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Limit to linear, quadratic, exponential, and absolute value functions.	F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
30	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maximums, and minimums. b. Graph general piecewise-defined functions, including step functions and absolute value functions. c. Graph exponential functions, showing intercepts and end behavior.	F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
31	Use the mathematical modeling cycle to solve real world problems involving linear, quadratic, exponential, absolute value, and general piecewise functions.		

32	Use mathematical and statistical reasoning with bivariate categorical data in order to draw conclusions and assess risk. Example: In a clinical trial comparing the effectiveness of flu shots A and B, 21 subjects in treatment group A avoided getting the flu while 29 contracted it. In group B, 12 avoided the flu while 13 contracted it. Discuss which flu shot appears to be more effective in reducing the chances of contracting the flu. Possible answer: Even though more people in group A avoided the flu than in group B, the proportion of people avoiding the flu in group B is greater than the proportion in group A, which suggests that treatment B may be more effective in lowering the risk of getting the flu. <table><tr><th></th><th>Contracted Flu</th><th>Did Not Contract Flu</th></tr><tr><td>Flu Shot A</td><td>28</td><td>21</td></tr><tr><td>Flu Shot B</td><td>13</td><td>12</td></tr><tr><td>Total</td><td>42</td><td>33</td></tr></table>		
33	Design and carry out an investigation to determine whether there appears to be an association between two categorical variables, and write a persuasive argument based on the results of the investigation. Example: Investigate whether there appears to be an association between successfully completing a task in a given length of time and listening to music while attempting the task. Randomly assign some students to listen to music while attempting to complete the task and others to complete the task without listening to music. Discuss whether students should listen to music while studying, based on that analysis.	S-CP.4	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. ★
34	Distinguish between quantitative and categorical data and between the techniques that may be for analyzing data of these two types. Example: The color of cars is categorical and so is summarized by frequency and proportion for each color category, while the mileage on each car's odometer is quantitative and can be summarized by the mean.		
35	Analyze the possible association between two categorical variables. a. Summarize categorical data for two categories in two-way frequency tables and represent using segmented bar graphs. b. Interpret relative frequencies in the context of categorical data (including joint, marginal, and conditional relative frequencies). c. Identify possible associations and trends in categorical data.	S-ID.5	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data. ★
36	Generate a two-way categorical table in order to find and evaluate solutions to real problems. a. Aggregate data from several groups to find an overall association between two categorical variables. b. Recognize and explore situations where the association between		

	two categorical variables is reversed when a third variable is considered (Simpson's Paradox). Example: In a certain city, Hospital 1 has a higher fatality rate than Hospital 2. But when considering mildly-injured patients and severely-injured patients as separate groups, Hospital 1 has a lower fatality rate among both groups than Hospital 2, since Hospital 1 is a Level 1 Trauma Center. Thus, Hospital 1 receives most of the severely injured patients who are less likely to survive overall but have a better chance of surviving in Hospital 1 than they would in Hospital 2.		
37	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events "or," "and," "not").	S-CP.1	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). ★
38	Explain whether two events, A and B, are independent, using two-way tables or tree diagrams.	S-CP.2	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent. ★
39	Compute the conditional probability of event A given event B, using two-way tables or tree diagrams.	S-CP.3	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. ★
40	Recognize and describe the concepts of conditional probability and independence in everyday situations and explain them using everyday language. Example: Contrast the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.	S-CP.5	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.★
41	Explain why the conditional probability of A given B is the fraction of B's outcomes that also belong to A, and interpret the answer in context. Example: the probability of drawing a king from a deck of cards, given that it is a face card, is (4/52)/(12/52), which is 1/3.	S-CP.6	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model. ★

A Side-by-Side Comparison of the Algebra II with Statistics Standards
in the
2019 Alabama Course of Study: Mathematics and the Common Core State Standards for Mathematics

	2019 Alabama Course of Study: Mathematics		Common Core State Standards for Mathematics
1	Identify numbers written in the form a + b*i*, where a and b are real numbers and $i^2 = -1$, as complex numbers. a. Add, subtract, and multiply complex numbers using the commutative, associative, and distributive properties.	N-CN.2	Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
2	Use matrices to represent and manipulate data.	N-VM.6	(+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
3	Multiply matrices by scalars to produce new matrices.	N-VM.7	(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
4	Add, subtract, and multiply matrices of appropriate dimensions.	N-VM.8	(+) Add, subtract, and multiply matrices of appropriate dimensions.
5	Describe the roles that zero and identity matrices play in matrix addition and multiplication, recognizing that they are similar to the roles of 0 and 1 in the real numbers.	N-VM.10	(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
6	Factor polynomials using common factoring techniques, and use the factored form of a polynomial to reveal the zeros of the function it defines.	A-SSE.3	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines. b.Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15t can be rewritten as (1.151/12)12t ≈ 1.01212t to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
7	Prove polynomial identities and use them to describe numerical relationships. Example: The polynomial identity $1 - x_n = (1 - x)(1 + x + x_2 + x_2 +\dots + x_{n-1} + x_n)$ can be used to find the sum of the first 11 terms of a geometric sequence with common ratio x by dividing both sides of the identity by (1 - x).	A-APR.4	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x2 + y2)2 = (x2 – y2)2 + (2xy)2 can be used to generate Pythagorean triples.
8	Explain why extraneous solutions to an equation may arise and how to check to be sure that a candidate solution satisfies an equation. Extend to radical equations.	A-REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
9	For exponential models, express as a logarithm the solution to $ab_{ct} =$ d, where a, c, and d are real numbers and the base b is 2 or 10; evaluate the logarithm using technology to solve an exponential equation.	F-LE.4	For exponential models, express as a logarithm the solution to ab^{ct}=d where a,c, and d are numbers and the base b is 2,10, or e; evaluate the logarithm using technology.

10	Create equations and inequalities in one variable and use them to solve problems. Extend to equations arising from polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions.	A-CED.1	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. ★
11	Solve quadratic equations with real coefficients that have complex solutions.	N-CN.7	Solve quadratic equations with real coefficients that have complex solutions.
12	Solve simple equations involving exponential, radical, logarithmic, and trigonometric functions using inverse functions.		
13	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales and use them to make predictions. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.	A-CED.2	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. ★
14	Explain why the x—coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x). a. Find the approximate solutions of an equation graphically, using tables of values, or finding successive approximations, using technology where appropriate. Extend to cases where f(x) and/or g(x) are polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions.	A-REI.11	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
15	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). Extend to polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions.	F-IF.9	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
16	Identify the effect on the graph of replacing f(x) by f (x) + k; k · f (x), f(k + x), and for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.	F-BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
17	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Note: Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries (including even and odd); end behavior; and periodicity. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.	F-IF.4	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★
18	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Extend to polynomial,	F-IF.5	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n

	trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.		engines in a factory, then the positive integers would be an appropriate domain for the function.★
19	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions.	F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
20	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Extend to polynomial, trigonometric (sine and cosine), logarithmic, reciprocal, radical, and general piecewise functions. a. Graph polynomial functions expressed symbolically, identifying zeros when suitable factorizations are available, and showing end behavior. b. Graph sine and cosine functions expressed symbolically, showing period, midline, and amplitude. c. Graph logarithmic functions expressed symbolically, showing intercepts and end behavior. d. Graph reciprocal functions expressed symbolically, identifying horizontal and vertical asymptotes. e. Graph square root and cube root functions expressed symbolically. f. Compare the graphs of inverse functions and the relationships between their key features, including but not limited to quadratic, square root, exponential, and logarithmic functions.	F-IF.7 A-APR.3	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
21	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle, building on work with non-right triangle trigonometry.	F-TF.2	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
22	Use the mathematical modeling cycle to solve real world problems involving polynomial, trigonometric (sine and cosine), logarithmic, radical, and general piecewise functions, from the simplification of the problem through the solving of the simplified problem, the interpretation of its solution, and the checking of the solution's feasibility.		
23	Use mathematical and statistical reasoning to measure confidence in order to draw conclusions and assess risk. Example: If candidate A is leading candidate B by 2% in a poll which has a margin of error of less than 3%, should we be surprised if candidate B wins the election?		
24	Design and carry out an experiment or survey to answer a question of interest, and write an informal persuasive argument based on the results.		

	Example: Use the statistical problem-solving cycle to answer the question, “Is there an association between playing a musical instrument and doing well in mathematics?”		
25	From a normal distribution, use technology to find the mean and standard deviation and estimate population percentages by applying the empirical rule. Explain why there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.	S-ID.4	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. ★
26	Describe the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. Examples: random assignment in experiments, random selection in surveys and observational studies	S-IC.3	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. ★
27	Distinguish between a statistic and a parameter and use statistical processes to make inferences about population parameters based on statistics from random samples from that population.	S-IC.1	Understand statistics as a process for making inferences about population parameters based on a random sample from that population. ★
28	Describe differences between randomly selecting samples and randomly assigning subjects to experimental treatment groups in terms of inferences drawn regarding a population versus regarding cause and effect. Example: Data from a group of plants randomly selected from a field allows inference regarding the rest of the plants in the field, while randomly assigning each plant to one of two treatments allows inference regarding differences in the effects of the two treatments. If the plants were both randomly selected and randomly assigned, we can infer that the difference in effects of the two treatments would also be observed when applied to the rest of the plants in the field.		
29	Explain the consequences of non-randomized assignment of subjects to groups in experiments. Example: Students are studying whether or not listening to music while completing mathematics homework improves their quiz scores. Rather than assigning students to either listen to music or not at random, they simply observe what the students do on their own and find that the music-listening group has a higher mean quiz score. Can they conclude that listening to music while studying is likely to raise the quiz scores of students who do not already listen to music? What other factors may have been responsible for the observed difference in mean quiz scores?		
30	Evaluate where bias, including sampling, response, or nonresponse bias, may occur in surveys, and whether results are representative of the population of interest. Example: Selecting students eating lunch in the cafeteria to participate in a survey may not accurately represent the student body, as students who do not eat in the cafeteria may not be		

	accounted for and may have different opinions, or students may not respond honestly to questions that may be embarrassing, such as how much time they spend on homework		
31	Evaluate the effect of sample size on the expected variability in the sampling distribution of a sample statistic. a. Simulate a sampling distribution of sample means from a population with a known distribution, observing the effect of the sample size on the variability. b. Demonstrate that the standard deviation of each simulated sampling distribution is the known standard deviation of the population divided by the square root of the sample size.		
32	Produce a sampling distribution by repeatedly selecting samples of the same size from a given population or from a population simulated by bootstrapping (resampling with replacement from an observed sample). Do initial examples by hand, then use technology to generate a large number of samples. a. Verify that a sampling distribution is centered at the population mean and approximately normal if the sample size is large enough. b. Verify that 95% of sample means are within two standard deviations of the sampling distribution from the population mean. c. Create and interpret a 95% confidence interval based on an observed mean from a sampling distribution.		
33	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. Example: Fifteen students are randomly assigned to a treatment group that listens to music while completing mathematics homework and another 15 are assigned to a control group that does not, and their means on the next quiz are found to be different. To test whether the differences seem significant, all the scores from the two groups are placed on index cards and repeatedly shuffled into two new groups of 15 each, each time recording the difference in the means of the two groups. The differences in means of the treatment and control groups are then compared to the differences in means of the mixed groups to see how likely it is to occur.	S-IC.5	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. ★
34	Define the radian measure of an angle as the constant of proportionality of the length of an arc it intercepts to the radius of the circle; in particular, it is the length of the arc intercepted on the unit circle.	G-C.5	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
		F-TF.1	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
35	Choose trigonometric functions (sine and cosine) to model periodic phenomena with specified amplitude, frequency, and midline.	F-TF.5	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★

36	Prove the Pythagorean identity $\sin^2(Ø) + \cos^2(Ø) = 1$ and use it to calculate trigonometric ratios.	F-TF.8	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$ and the quadrant of the angle.
37	Derive and apply the formula A = 1/2 *ab*sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side, extending the domain of sine to include right and obtuse angles.	G-SRT.9	(+) Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
38	Derive and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. Extend the domain of sine and cosine to include right and obtuse angles. Examples: surveying problems, resultant forces	G-SRT.11	(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

A Side-by-Side Comparison of the Mathematical Modeling Standards
in the
2019 Alabama Course of Study: Mathematics and the Common Core State Standards for Mathematics

	2019 Alabama Course of Study: Mathematics		Common Core State Standards for Mathematics
1	Use the full mathematical modeling or statistical problem-solving cycle to answer a real- world problem of particular student interest, incorporating standards from across the course. Examples: Use a mathematical model to design a three-dimensional structure and determine whether particular design constraints are met; to decide under what conditions the purchase of an electric vehicle will save money; to predict the extent the level of the ocean will rise due to the melting polar ice caps; or to interpret the claims of a statistical study regarding the economy.		
2	Use elements of the Mathematical Modeling Cycle to solve real-world problems involving finances.		
3	Organize and display financial information using arithmetic sequences to represent simple interest and straight-line depreciation.		
4	Organize and display financial information using geometric sequences to represent compound interest including periodic (yearly, monthly, weekly) and continuous compounding. a. Explain the relationship between annual percentage yield (APY) and annual percentage rate (APR) as values for r in the formulas $A=P(1+r)^t$ and $A=Pe^{rt}$.		
5	Compare simple and compound interest, and straight-line and proportional depreciation.		
6	Investigate growth and reduction of credit card debt using spreadsheets, including variables such as beginning balance, payment structures, credits, interest rates, new purchases, finance charges, and fees.		
7	Compare and contrast housing options including renting, leasing to purchase, purchasing with a mortgage, and purchasing with cash. a. Research and evaluate various mortgage products available to consumers. b. Compare monthly mortgage payments for different terms, interest rates, and down payments. c. Analyze the financial consequence of buying a home (mortgage payments vs. potentially increasing resale value) versus investing the money saved when renting, assuming that renting is the less expensive option.		
8	Investigate the advantages and disadvantages of various means of acquiring an automobile, including leasing, purchasing by cash, and purchasing by loan.		

9	Use the Mathematical Modeling Cycle to solve real-world problems involving the design of three-dimensional objects.		
10	Construct a two-dimensional visual representation of a three-dimensional object or structure. a. Determine the level of precision and the appropriate tools for taking the measurements in constructing a two-dimensional visual representation of a three-dimensional object or structure. b. Create an elevation drawing to represent a given solid structure, using technology where appropriate. c. Determine which measurements cannot be taken directly and must be calculated based on other measurements when constructing a two-dimensional visual representation of a three-dimensional object or structure. d. Determine an appropriate means to visually represent an object or structure, such as drawings on paper or graphics on computer screens.		
11	Plot coordinates on a three-dimensional Cartesian coordinate system and use relationships between coordinates to solve design problems. a. Describe the features of a three-dimensional Cartesian coordinate system and use them to graph points. b. Graph a point in space as the vertex of a right prism drawn in the appropriate octant with edges along the x, y, and z axes. c. Find the distance between two objects in space given the coordinates of each. Examples: Determine whether two aircraft are flying far enough apart to be safe; find how long a zipline cable would need to be to connect two platforms at different heights on two trees. d. Find the midpoint between two objects in space given the coordinates of each. Example: If two asteroids in space are traveling toward each other at the same speed, find where they will collide.		
12	Use technology and other tools to explore the results of simple transformations using three-dimensional coordinates, including translations in the x, y, and/or z directions; rotations of 90° or 180° about the x, y, and z axes; reflections over the xy, yz, and xy planes; and dilations from the center. _ Example: Given the coordinates of the corners of a room in a house, find the coordinates of the same room facing a different direction.		
13	Create a scale model of a complex three-dimensional structure based on observed measurements and indirect measurements, using translations, reflections, rotations, and dilations of its components. Example: Develop a plan for a bridge structure using geometric properties of its parts to determine unknown measures and represent the plan in three dimensions.		

14	Use elements of the Mathematical Modeling Cycle to make predictions based on measurements that change over time, including motion, growth, decay, and cycling.		
15	Use regression with statistical graphing technology to determine an equation that best fits a set of bivariate data, including nonlinear patterns, carbon dating measurements, online streaming viewership a. Create a scatter plot with a sufficient number of data points to predict a pattern. b. Describe the overall relationship between two quantitative variables (increase, decrease, linearity, concavity, extrema, inflection) or pattern of change. c. Make a prediction based upon patterns.		
16	Create a linear representation of non-linear data and interpret solutions, using technology and the process of linearization with logarithms.		
17	Use the Statistical Problem Solving Cycle to answer real-world questions.		
18	Construct a probability distribution based on empirical observations of a variable. Example: Record the number of student absences in class each day and find the probability that each number of students will be absent on any future day. a. Estimate the probability of each value for a random variable based on empirical observations or simulations, using technology. b. Represent a probability distribution by a relative frequency histogram and/or a cumulative relative frequency graph. c. Find the mean, standard deviation, median, and interquartile range of a probability distribution and make long-term predictions about future possibilities. Determine which measures are most appropriate based upon the shape of the distribution.		
19	Construct a sampling distribution for a random event or random sample. Examples: How many times do we expect a fair coin to come up "heads " in 100 flips, and on average how far away from this expected value do we expect to be on a specific set of flips? What do we expect to be the average height for a random sample of students in a local high school given the mean and standard deviation of the heights of all students in the high school? a. Use the binomial theorem to construct the sampling distribution for the number of successes in a binary event or the number of positive respondents to a yes/no question in a random sample. b. Use the normal approximation of a proportion from a random event or sample when conditions are met.		

	c. Use the central limit theorem to construct a normal sampling distribution for the sample mean when conditions are met. d. Find the long-term probability of a given range of outcomes from a random event or random sample.		
20	Perform inference procedures based on the results of samples and experiments. a. Use a point estimator and margin of error to construct a confidence interval for a proportion or mean. b. Interpret a confidence interval in context and use it to make strategic decisions. Example. short-term and long-term budget predictions for a business c. Perform a significance test for null and alternative hypotheses. d. Interpret the significance level of a test in the context of error probabilities, and use the results to make strategic decisions. Example: How do you reduce the rate of human error on the floor of a manufacturing plant?		
21	Critique the validity of reported conclusions from statistical studies in terms of bias and random error probabilities.		
22	Conduct a randomized study on a topic of student interest (sample or experiment) and draw conclusions based upon the results. Example: Record the heights of thirty randomly selected students at your high school. Construct a confidence interval to estimate the true average height of students at your high school. Question whether or not this data provides significant evidence that your school 's average height is higher than the known national average, and discuss error probabilities.		

A Side-by-Side Comparison of the Finite Mathematics Standards
in the
2019 Alabama Course of Study: Mathematics and the Common Core State Standards for Mathematics

	2019 Alabama Course of Study: Mathematics		Common Core State Standards for Mathematics
1	Represent logic statements in words, with symbols, and in truth tables, including conditional, biconditional, converse, inverse, contrapositive, and quantified statements.		
2	Represent logic operations such as and, or, not, nor, and x or (exclusive or) in words, with symbols, and in truth tables.		
3	Use truth tables to solve application-based logic problems and determine the truth value of simple and compound statements including negations and implications. a. Determine whether statements are equivalent and construct equivalent statements. Example: Show that the contrapositive of a statement is logically equivalent.		
4	Determine whether a logical argument is valid or invalid, using laws of logic such as the law of syllogism and the law of detachment. a. Determine whether a logical argument is a tautology or a contradiction.		
5	Prove a statement indirectly by proving the contrapositive of the statement.		
6	Use multiple representations and methods for counting objects and developing more efficient counting techniques. Note: Representations and methods may include tree diagrams, lists, manipulatives, overcounting methods, recursive patterns, and explicit formulas.		
7	Develop and use the Fundamental Counting Principle for counting independent and dependent events. a. Use various counting models (including tree diagrams and lists) to identify the distinguishing factors of a context in which the Fundamental Counting Principle can be applied. Example: Apply the Fundamental Counting Principle in a context that can be represented by a tree diagram in which there are the same number of branches from each node at each level of the tree.		
8	Using application-based problems, develop formulas for permutations, combinations, and combinations with repetition and compare student-derived formulas to standard representations of the formulas. Example: If there are r objects chosen from n objects, then the number of permutations can be found by the product [n(n—l) . . . (n-r)(n-r+1)] as compared to the standard formula n!/(n-r)!.		

	a. Identify differences between applications of combinations and permutations. b. Using application-based problems, calculate the number of permutations of a set with n elements. Calculate the number of permutations of r elements taken from a set of n elements. c. Using application-based problems, calculate the number of subsets of size r that can be chosen from a set of n elements, explaining this number as the number of combinations “n choose r.” d. Using application-based problems, calculate the number of combinations with repetitions of r elements from a set of n elements as (n + r -1) choose r.		
9	Use various counting techniques to determine probabilities of events.		
10	Use the Pigeonhole Principle to solve counting problems.		
11	Find patterns in application problems involving series and sequences, and develop recursive and explicit formulas as models to understand and describe sequential change. Examples: fractals, population growth		
12	Determine characteristics of sequences, including the Fibonacci Sequence, the triangular numbers, and pentagonal numbers. Example. Write a sequence of the first 10 triangular numbers and hypothesize a formula to find the nth triangular number.	F-IF.3	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.
13	Use the recursive process and difference equations to create fractals, population growth models, sequences, and series.		
14	Use mathematical induction to prove statements involving the positive integers. Examples: Prove that 3 divides $2^{2n} - 1$ for all positive integers n; prove that $I + 2 + 3 + \ldots + n = n(n + 1)/2$; prove that a given recursive sequence has a closed form expression.		
15	Develop and apply connections between Pascal’s Triangle and combinations.		
16	Use vertex and edge graphs to model mathematical situations involving networks. a. Identify properties of simple graphs, complete graphs, bipartite graphs, complete bipartite graphs, and trees.		
17	Solve problems involving networks through investigation and application of existence and nonexistence of Euler paths, Euler circuits, Hamilton paths, and Hamilton circuits. Note: Real world contexts modeled by graphs may include roads or communication networks. Example: show why a 5x5 grid has no Hamilton circuit. a. Develop optimal solutions of application-based problems using existing and student-created algorithms. b. Give an argument for graph properties.		

	Example: Explain why a graph has a Euler cycle if and only if the graph is connected and ever vertex has even degree. Show that any tree with n vertices has n — 1 edges.		
18	Apply algorithms relating to minimum weight spanning trees, networks, flows, and Steiner trees. Example: traveling salesman problem a. Use shortest path techniques to find optimal shipping routes. b. Show that every connected graph has a minimal spanning tree. 0. Use Kruskal's Algorithm and Prim's Algorithm to determine the minimal spanning tree of a weighted graph.		
19	Use vertex-coloring, edge-coloring, and matching techniques to solve application-based problems involving conflict. Examples: Use graph-coloring techniques to color a map of the western states of the United States so that no adjacent states are the same color, determining the minimum number of colors needed and why no fewer colors may be used; use vertex colorings to determine the minimum number of zoo enclosures needed to house ten animals given their cohabitation constraints; use vertex colorings to develop a time table for scenarios such as scheduling club meetings or for housing hazardous chemicals that cannot all be safely stored together in warehouses.		
20	Determine minimum time to complete a project using algorithms to schedule tasks in order, including critical path analysis, the list-processing algorithm, and student-created algorithms.		
21	Use the adjacency matrix of a graph to determine the number of walks of length n in a graph.		
22	Analyze advantages and disadvantages of different types of ballot voting systems. a. Identify impacts of using a preferential ballot voting system and compare it to single candidate voting and other voting systems. b. Analyze the impact of legal and cultural features of political systems on the mathematical aspects of elections. Examples: mathematical disadvantages of third parties, the cost of run-of elections		
23	Apply a variety of methods for determining a winner using a preferential ballot voting system, including plurality, majority, run-off with majority, sequential run-off with majority, Borda count, pairwise comparison, Condorcet and approval voting.		
24	Identify issues of fairness for different methods of determining a winner using a preferential voting ballot and other voting systems and identify paradoxes that can result. Example: Arrow 's Theorem		
25	Use methods of weighted voting and identify issues of fairness related to weighted voting.		

	Example: determine the power of voting bodies using the Banzhaf power index a. Distinguish between weight and power in voting.		
26	Explain and apply mathematical aspects of fair division, with respect to classic problems of apportionment, cake cutting, and estate division. Include applications in other contexts and modern situations		
27	Identify and apply historic methods of apportionment for voting districts including Hamilton, Jefferson, Adams, Webster, and Huntington-Hill. Identify issues of fairness and paradoxes that may result from methods. Examples: the Alabama paradox, population paradox		
28	Use spreadsheets to examine apportionment methods in large problems. Example: apportion the 435 seats in the US. House of Representatives using historically applied methods		
29	Critically analyze issues related to information processing including accuracy, efficiency, and security.		
30	Apply ciphers (encryption and decryption algorithms) and cryptosystems for encrypting and decrypting including symmetric-key or public-key systems. a. Use modular arithmetic to apply RSA (Rivest-Shamir-Adleman) public-key cryptosystems. b. Use matrices and their inverses to encode and decode messages.		
31	Apply error-detecting codes and error-correcting codes to determine accuracy of information processing.		
32	Apply methods of data compression. Example: Hufinan codes		

A Side-by-Side Comparison of the Precalculus Standards
in the
2019 Alabama Course of Study: Mathematics and the Common Core State Standards for Mathematics

	2019 Alabama Course of Study: Mathematics		Common Core State Standards for Mathematics
1	Define the constant *e* in a variety of contexts. Example: the total interest earned if a 100% annual rate is continuously compounded. a. Explore the behavior of the function $y=e^x$ and its applications. b. Explore the behavior of ln(x), the logarithmic function with base *e*, and its applications.		
2	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.	N-CN.3	(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
3	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.	N-CN.4	(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
4	Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. Example: $(-1 + \sqrt{3}i)^3 = 8$ because $(-1 + \sqrt{3}i)$ has modulus 2 and argument 120°.	N-CN.5	(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, (–1 + √3 i)3 = 8 because (–1 + √3 i) has modulus 2 and argument 120°.
5	Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.	N-CN.6	(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.
6	Analyze possible zeros for a polynomial function over the complex numbers by applying the Fundamental Theorem of Algebra, using a graph of the function, or factoring with algebraic identities.		
7	Determine numerically, algebraically, and graphically the limits of functions at specific values and at infinity. a. Apply limits of functions of functions at specific values and at infinity in problems involving convergence and divergence.		
8	Explain that vector quantities have both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes. Examples: v, |v|, ||v||, v.	N-VM.1	(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).
9	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.	N-VM.2	(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
10	Solve problems involving velocity and other quantities that can be represented by vectors.	N-VM.3	(+) Solve problems involving velocity and other quantities that can be represented by vectors.
11	Find the scalar (dot) product of two vectors as the sum of the products of corresponding components and explain its relationship to the cosine of the angle formed by two vectors.		

12	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule, understanding that the magnitude of a sum of two vectors is not always the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Explain vector subtraction, v - w, as v + (-w), where -w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.	N-VM.4	(+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
13	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by sealing vectors and possibly reversing their direction; perform scalar multiplication component-wise. Example: $c(v_x, v_y) = (cv_x, cv_y)$ b. Compute the magnitude of a scalar multiple cv using ||cv|| = lclv. Compute the direction of cv knowing that when |c| V ≠ 0, the direction of cv is either along v (for c > 0) or against v (for c < 0).	N-VM.5	(+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple cv using ||cv|| = |c|v. Compute the direction of cv knowing that when |c|v ≠ 0, the direction of cv is either along v (for c > 0) or against v (for c < 0).
14	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.	N-VM.11	(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
15	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), use the formula to solve problems, extending to infinite geometric series. Examples: calculate mortgage payments; determine the long-term level of medication if a patient takes 50 mg of a medication every 4 hours, while 70% of the medication is filtered out of the patient's blood.	A-SSE.4	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.★
16	Derive and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x - a is p(a), so p(a) = 0 if and only if (x — a) is a factor of p(x).	A-APR.2	Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
17	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer, n, where x and y are any numbers.	A-APR.5	(+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
18	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated cases, a computer algebra system.	A-APR.6	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
19	Add, subtract, multiply, and divide rational expressions. a. Explain why rational expressions form a system analogous to the	A-APR.7	(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction,

	rational numbers, which is closed under addition, subtraction, multiplication, and division by a nonzero rational expression.		multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
20	Explain each step in solving an equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a clear-cut solution. Construct a viable argument to justify a solution method. Include equations that may involve linear, quadratic, polynomial, exponential, logarithmic, absolute value, radical, rational, piecewise, trigonometric functions, and their inverses.	A-REI.1	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
21	Solve simple rational equations in one variable, and give examples showing how extraneous solutions may arise.	A-REI.2	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
22	Represent a system of linear equations as a single matrix equation in a vector variable.	A-REI.8	(+) Represent a system of linear equations as a single matrix equation in a vector variable.
23	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater).	A-REI.9	(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).
24	Compare and contrast families of functions and their representations (algebraically, numerically, and verbally in terms of their key features. Note: Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; maximums and minimums; symmetries (including even and odd); end behavior; asymptotes; and periodicity. Families of functions include but are not limited to linear, quadratic, polynomial, exponential, logarithmic, absolute value, radical, rational, piecewise, trigonometric, and their inverses.		
25	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Extend from polynomial, exponential, logarithmic, and radical to rational and all trigonometric functions. a. Find the difference quotient $(f(x+\Delta x) - f(x))/\Delta x$ of a function and use it to evaluate the average rate of change at a point. b. Explore how the average rate of change of a function over an interval (presented symbolically or as a table) can be used to approximate the instantaneous rate of change at a point as the interval decreases.	F-IF.6	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
26	Graph functions expressed symbolically and show key features of the graph, by hand and using technology. Use the equation of functions to identify key features in order to generate a graph. a. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. b. Graph trigonometric functions and their inverses, showing period, midline, amplitude, and phase shift.	F-IF.7	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

			c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
27	Compose functions. Extend to polynomial, trigonometric, radical, and rational functions. Example: If T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T (h(t)) is the temperature at the location of the weather balloon as a function of time.		
28	Find inverse functions. a. Given that a function has an inverse, write an expression for the inverse of the function. Example: Given f(x) =2 x_3 or f(x) = (x + l)/(x - l) for x ≠1 find f_{-1}(x). b. Verify by composition that one function is the inverse of another. c. Read values of an inverse function from a graph or a table, given that the function has an inverse. d. Produce an invertible function from a non-invertible function by restricting the domain.	F-BF.4	Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x_3 or f(x) = (x + l)/(x - l) for x ≠1. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain.
29	Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents. Extend from logarithms with base 2 and 10 to a base of *e*.	F-BF.5	(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
30	Identify the effect on the graph of replacing f (x) by f (x) + k, k • f (x), f (k • x), and f (x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Extend the analysis to include all trigonometric, rational, and general piecewise-defined functions with and without technology. Example: Describe the sequence of transformations that will relate y=sin(x) and y=2sin(3 x).	F-BF.3	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
31	Graph conic sections from second-degree equations, extending from circles and parabolas to ellipses and hyperbolas, using technology to discover patterns. a. Graph conic sections given their standard form. Example: The graph of $\frac{x^2}{9}+\frac{(y-3)^2}{4}=1$ will be an ellipse centered at (0,3) with major axis 3 and minor axis 2, while the graph of $\frac{x^2}{9}-\frac{(y-3)^2}{4}=1$ will be a hyperbola centered at (0,3) with asymptotes with slope ±3/2.		

	b. Identify the conic section that will be formed, given its equation in general form. Example: $5y^2 - 25x^2 = -25$ will be a hyperbola.		
32	Solve application-based problems involving parametric and polar equations. a. Graph parametric and polar equations. b. Convert parametric and polar equations to rectangular form.		
33	Use special triangles to determine geometrically the values of sine, cosine, and tangent for $\pi/3$, $\pi/4$, and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x, and 2 π – x in terms of their values for x, where x is any real number.	F-TF.3	(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–x, π+x, and 2π–x in terms of their values for x, where x is any real number.
34	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.	F-TF.4	(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
35	Demonstrate that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.	F-TF.6	(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
36	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.	F-TF.7	(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★
37	Use trigonometric identities to solve problems. a. Use the Pythagorean identity $\sin^2(\emptyset) + \cos^2(\emptyset) = 1$ to derive the other forms of the identity. Example: $1 + \cot^2(\emptyset) = \csc^2(\emptyset)$ b. Derive and use the double angle formulas for sine, cosine, and tangent. c. Use the angle sum formulas for sine, cosine, and tangent to derive the double angle formulas. d. Use the Pythagorean and double angle identities to prove other simple identities.		

Common Core State Standards for Mathematics
Not Matched with the
2019 Alabama Course of Study: Mathematics

			Common Core State Standards for Mathematics
			Number and Quantity
		N-RN.3	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.
		N-CN.8	(+) Extend polynomial identities to the complex numbers. For example, rewrite x2 + 4 as (x + 2i)(x – 2i).
		N-CN.9	(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
		N-VM.9	(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
		N-VM.12	(+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.
			Algebra
		A-REI.3	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
			Functions
		F-IF.8	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)t, y = (0.97)t, y = (1.01)12t, y = (1.2)t/10, and classify them as representing exponential growth or decay.
		F-BF.2	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★
		F-TF.9	(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
			Geometry

		G-CO.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
		G-CO.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.
		G-C.1	Prove that all circles are similar.
		G-C.3	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
		G-C.4	(+) Construct a tangent line from a point outside a given circle to the circle.
		G-GPE.2	Derive the equation of a parabola given a focus and directrix.
		G-GPE.3	(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
		G-GPE.4	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).
		G-GPE.6	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
		G-MG.2	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★
			Statistics and Probability
		S-IC.2	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model? ★
		S-IC.4	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. ★
		S-IC.6	Evaluate reports based on data. ★
		S-CP.7	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. ★
		S-CP.8	(+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. ★
		S-CP.9	(+) Use permutations and combinations to compute probabilities of compound events and solve problems. ★
		S-MD.1	(+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the

			corresponding probability distribution using the same graphical displays as for data distributions. ★
		S-MD.2	(+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution. ★
		S-MD.3	(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes. ★
		S-MD.4	(+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?★
		S-MD.5	(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. ★ a. Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant. b. Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.
		S-MD.6	(+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator). ★
		S-MD.7	(+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game). ★
	Note: Some matches are exact matches while others are partial matches or conceptual matches.		