Review of 2019 Alabama Course of Study: Mathematics

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This is a review of the 2019 Alabama Course of Study: Mathematics (2019 AL standards or 2019 standards). This review was conducted by reading the standards and matching them side by side with the Common Core State Standards for Mathematics (CCSS). Reviewed were the K-8 regular standards. The standards for Grade 7 Accelerated and Grade 8 Accelerated were not reviewed.

The introductory information for the standards includes the list of people involved in generating the Course of Study document. It clearly states the document was developed by them and makes no claim they wrote the standards. It is wise no such claim is made since from this review, it can be concluded that the standards are the CCSS standards with some edits and revisions. The edits and revisions made have not rendered the standards significantly different from the CCSS to credibly say they were written by Alabama educators and other professionals within the state and not by the CCSS authors.

Changes that are noted as insignificant are ones that have not changed the substance or content and likely will not be subject to different interpretations by different practitioners in the classroom. If a change is not noted as insignificant or significant, it is likely an insignificant change. Changes noted as significant are ones that change the meaning or content of the standard or open them up to inconsistent and differing interpretations by practitioners resulting in different classroom instruction. Significant changes may improve or strengthen a standard or may weaken a standard.

This review starts with a comparison of how the standard algorithm is addressed in the Common Core State Standards (CCSS), the 2016 Revised Alabama Course of Study for Mathematics, and the 2019 Alabama Course of Study: Mathematics. Following that, there are sections that address Standards for Mathematical Practice, Review/Comparison of Grade 2 Standards, Comments of Review/Comparison of Grades K-8, Fluency, and Concluding Comments.

Standard Algorithm

Addition and Subtraction Grade 2

Common Core State Standards
Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.
2016 Revised Alabama Course of Study for Mathematics
Fluently add and subtract within 100 using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction. [2-NBT5]
2019 Alabama Course of Study: Mathematics
Fluently add and subtract within 100, using strategies based on place value, properties of operations, and/or the relationship between addition and subtraction.

These three standards are identical with the exception of the comma added in the 2019 standard. The standard algorithm for addition and subtraction is not mentioned or required in grade 2. Students at this grade level are capable of learning and employing the standard algorithm for addition and subtraction.

Addition and Subtraction Grade 4

Common Core State Standards
Fluently add and subtract multi-digit whole numbers using the standard algorithm.
2016 Revised Alabama Course of Study for Mathematics
Fluently add and subtract multi-digit whole numbers using the standard algorithm. [4-NBT4]
2019 Alabama Course of Study: Mathematics
Use place value strategies to fluently add and subtract multi-digit whole numbers and connect strategies to the standard algorithm.

The CCSS and 2016 are identical and both require the use of the standard algorithm for adding and subtracting. This is an important requirement. Unfortunately, waiting until grade 4 to require this is an unnecessary delay for something that should be taught and required in grade 2. The 2019 standard does not require the use of the standard algorithm. At best, it only requires students to have enough awareness of the standard algorithm to connect strategies to it. This is a significant change that substantially weakens the whole set of standards.

Multiplication Grade 3

Common Core State Standards
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers.
2016 Revised Alabama Course of Study for Mathematics
Fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that 8 × 5 = 40, one knows 40 ÷ 5 = 8) or properties of operations. By the end of Grade 3, know from memory all products of two one-digit numbers. [3-OA7]
2019 Alabama Course of Study: Mathematics
Use strategies based on properties and patterns of multiplication to demonstrate fluency with multiplication and division within 100. a. Fluently determine all products obtained by multiplying two one-digit numbers.

The CCSS and 2016 standards are identical. None of the sets mention requiring the use of the standard algorithm for multiplication in grade 3. The 2019 standard appears to be a weakened rearrangement of the wording of the CCSS and 2016 standards. This is a significant change rendering the standard weaker because it emphasizes “use strategies” by leading with that in the standard rather than leading with “fluently multiply”. This change shifts the focus from math to pedagogy.

Multiplication Grade 5

Common Core State Standards
Fluently multiply multi-digit whole numbers using the standard algorithm.
2016 Revised Alabama Course of Study for Mathematics
Fluently multiply multi-digit whole numbers using the standard algorithm. [5-NBT5]
2019 Alabama Course of Study: Mathematics
Fluently multiply multi-digit whole numbers using the standard algorithm.

All three standards are identical and require the use of the standard algorithm. This is an important requirement. Unfortunately, waiting until grade 5 to require this is an unnecessary delay for something that should be taught and required in grade 3.

Division Grade 4

Common Core State Standards
Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.
2016 Revised Alabama Course of Study for Mathematics
Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations, and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models. [4-NBT6]
2019 Alabama Course of Study: Mathematics
Use strategies based on place value, properties of operations, and/or the relationship between multiplication and division to find whole-number quotients and remainders with one-digit divisors and up to four-digit dividends. a. Illustrate and explain quotients using equations, rectangular arrays, and/or area models.

The CCSS and 2016 standards are identical. None of the sets mention or require the use of the standard algorithm for division in grade 4. The 2019 standard appears to be a weakened rearrangement of the wording of the CCSS and 2016 standards. This is a significant change rendering the standard weaker because it emphasizes “use strategies” by leading with that in the standard rather than leading with “find whole-number quotients”. This change shifts the focus from math to pedagogy

Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors. This is what we want students to be able to do. The rest is pedagogy. To change things around and lead with the verb “use,” indicate the emphasis and importance of the standard is on Use strategies based on place value, properties of operations, and/or the relationship between multiplication and division.

Division Grade 6

Common Core State Standards
Fluently divide multi-digit numbers using the standard algorithm.
2016 Revised Alabama Course of Study for Mathematics
Fluently divide multi-digit numbers using the standard algorithm. [6-NS2]
2019 Alabama Course of Study: Mathematics
Fluently divide multi-digit whole numbers using the standard algorithm.

These three standards are identical with the exception of the word "whole" being added in the 2019 standard. All three require the use of the standard algorithm for division. Unfortunately, waiting until grade 6 to require this is an unnecessary delay for something that should be taught and required in grade 4.

Add, subtract, multiply, and divide multi-digit decimals Grade 6

Common Core State Standards
Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation.
2016 Revised Alabama Course of Study for Mathematics
Fluently add, subtract, multiply, and divide multi-digit decimals using the standard algorithm for each operation. [6-NS3]
2019 Alabama Course of Study: Mathematics
Add, subtract, multiply, and divide decimals using the standard algorithms.

The CCSS and 2016 standards are identical. The significant change to the 2019 standard appears to make it a weaker standard because it doesn't require fluency or dividing multi-digit decimals. It is interesting that the 2019 standard does not require students to use the standard algorithm for adding whole numbers (see grade 4 comments) but does require the use when adding decimals.

The 2019 standards call on students to learn and use strategies based on place value but does not require the use of the standard algorithm for addition and subtraction. While the use of the standard algorithm is required for multiplication and division, that requirement comes as a two-year delay when compared to requirements in high performing countries. This will place Alabama students exiting sixth grade two years behind their international peers. Issues related to the use of strategies based on place value and the two year delay are addressed in a paper titled Common Does Not Equal Excellence. The delayed requirement related to the standard algorithm for multiplication and division and the absence of a requirement for the use of the standard algorithm for addition and subtraction prior to grade 6 means successful achievement in math for most students will be doomed as they move into higher grades.

Standards for Mathematical Practice

The 2019 standard set has eight Standards for Mathematical Practice as the CCSS and 2016 set. In the 2019 standards they are called Student Mathematical Practices. The descriptive narratives for each of the eight Student Mathematical Practices are in essence the same as the narratives in the CCSS and are identical to the 2016 narratives with the

exception of few insignificant word changes. There are many sections, statements, and phrases identical to the CCSS. Where there are changes in the wording, they are insignificant.

Review/Comparison of Grade 2 Standards

The 2019 Grade 2 standards were selected as a grade level set representative of the K-8 standards for close examination.

The domains and clusters in the grade 2 overview of the CCSS and 2016 standards are identical. The 2019 domains and clusters have a separate domain for Data Analysis and Measurement instead of being combined as in the CCSS and 2016. The clusters are identical in all three with the exception of 2019 saying “Collect and analyze data and interpret results” instead of “Represent and interpret data” as in the CCSS and 2016 set.

A Side-by-Side Comparison of the Second Grade Standards in the 2019 Alabama Course of Study: Mathematics and the Common Core State Standards for Mathematics has been prepared to provide a visual showing these standards are clearly the Common Core State Standards with some revisions. The 2019 standards contain one standard not found in the CCSS. Standard 24 provides more explicit detail in what is expected. Overall, the revisions and added standard are insignificant and neither improve nor weaken the base CCSS standards. As a result, any strength or weakness of the CCSS apply to the 2019 standards at this grade level.

Comments of Review/Comparison of Grades K-8

Kindergarten, Grade 1, Grade 3

- These are basically the same standards as the CCSS. In many cases they are identical. Some are CCSS standards with few insignificant revisions. A few CCSS standards have been rewritten and appear to be clearer and more easily understood. These standards retain the pedagogy embedded in the CCSS.

Grade 1

- A note for Standard 6 is addressed in the Fluency section of this review.
- Standard 9 is an added standard that is not included in the CCSS. It is identical to the standard added in Grade 2.
- Standard 20 is an added standard related to money/coins.

Grade 3

- Standard 5 calls for students to develop properties of operations for multiplying and dividing.
- Standard 7 does not require knowing from memory products of two one-digit numbers as the CCSS standard does. This considerably weakens this set of standards and serves as further indication fluency in these standards, as defined in grade 1, means procedural and computational fluency and not fact fluency.
- Standard 18 requires students to tile, but not to multiply side lengths, in order to find the area of a triangle. The CCSS requires tiling and multiplying side lengths. This

significantly weakens the standards for this grade level and necessitates students being assessed on their ability to tile. The inclusion of tiling in a standard is pedagogy. The standard would stand well to simply read, "Find the area of a rectangle."

Grade 4

- A number of the 2019 standards are the same as corresponding CCSS standards but are written in a clearer manner with some having slightly different formatting, often making them easy to understand. Although some standards have had some pedagogy stripped from them, for the most part, the pedagogy required by the CCSS is intact in the 2019 standards.
- Standard 9 reads, "Use place value strategies to fluently add and subtract multi-digit whole numbers and connect strategies to the standard algorithm." This is addressed in the Standard Algorithm section of this review.

Grades 5, 6, 7, and 8

- These are easily identifiable CCSS standards with some revisions.
- In Grade 6, it appears that more standards have been rewritten but are basically similar to or the same as the CCSS in terms of content.
- In Grade 8, many CCSS standards have been rewritten in a manner that nicely streamlines them and strips out some pedagogy while at the same time weakens them in terms of content and student learning expectation.

The Grade 7 Accelerated and Grade 8 Accelerated standards were not reviewed.

Fluency

Standard 12 in the 2019 AL standards says, "Fluently add and subtract within 5." This leads to an interest in examining what these standards mean by fluency. Consider three types of fluency in math, math fact fluency, procedural fluency, and computational fluency. Here are some readily available, and common, definitions for each:

Math Fact Fluency.

- Fluency means how fast a person can retrieve math facts to working memory from storage memory. This retrieval should take only nano seconds or micro seconds. (Find here.)
- Math fact fluency is the ability to recall the answers to basic math facts automatically and without hesitation. (Find here.)
- Students achieve math fact fluency through automaticity, or the ability to deliver a correct answer immediately from memory without relying on calculation. (Find here.)

Procedural Fluency.

- Procedural fluency is the ability to apply procedures accurately, efficiently, and flexibly; to transfer procedures to different problems and contexts; to build or modify procedures from other procedures; and to recognize when one strategy or procedure is more appropriate to apply than another. (Find here.)
- procedural fluency (skill in carrying out procedures flexibly, accurately, efficiently and appropriately) (Found in the Standards for Mathematical Practice section of the

> CCSS and in the 2019 AL standards Student Mathematical Practices, AL added one comma)

Computational Fluency

- Students exhibit computational fluency when they demonstrate flexibility in the computational methods they choose, understand and can explain these methods, and produce accurate answers efficiently. (Find here.)
- Computational fluency refers to having efficient and accurate methods for computing. (Find here.)

Procedural and computational fluency are very similar and neither requires math fact fluency.

A note for Grade 1 Standard 6 of the 2019 Alabama standards reads:

- Note: Fluency involves a mixture of "just knowing" answers, knowing answers from patterns, and knowing answers from the use of strategies. The word fluently is used in the standards to mean accurately, efficiently, and flexibly.

The first sentence holds hope that fact fluency and memorization will be required while the second sentence removes any such hope. The detailed pedagogy in Standard 6 does not call for fact fluency, rather it develops procedural fluency and strategies for finding facts.

The 2019 standards stay true their use of the word fluently to mean accurately, efficiently, and flexibly. Fluently, as used in these standards, means procedural/computational fluency. It is not used in any way to mean memorization or automatic recall of basic math facts.

Grade 2, Standard 2 reads:

> Fluently add and subtract with 20 using mental strategies such as counting on, making ten, decomposing a number leading to ten, using the relationship between addition and subtraction, and creating equivalent but easier or known sums.
>
> a. State automatically all sums of two one-digit numbers.

The first part of the standard provides the procedural strategies students should become fluent with and does not call for math fact fluency. The second part of the standard, "a. State automatically all sums of two one-digit numbers.," does call for math fact fluency without using the term "fluency" or "fluently". There is not a similar standard requiring math fact fluency for multiplication.

If one chooses to view procedural/computational fluency to include fact fluency, then standards should be included developing and requiring math fact fluency. The 2019 AL standards do develop and require math fact fluency for sums of two one-digit numbers but not for products of two one-digit numbers as required by the CCSS.

Concluding Comments

The K-8 2019 Alabama Course of Study: Mathematics standards are one and the same as the Common Core State Standards. Many standards are identical. Ones that aren't identical have only slight insignificant changes made to them. Overall, such changes did not alter the content. The pedagogy in the CCSS remains intact in the new AL standards. If one has looked at and compared numerous sets of pre-CCSS math standards with the CCSS, it becomes evident the CCSS standards are uniquely written and easy to recognize when the CCSS is the genesis for a "new" set of standards. There were a few changes to CCSS standards that renders the 2019 AL set slightly weaker. These standards are not independent of the CCSS.

Anyone making such a claim that these are "Alabama standards written by educators and other professionals from around the state" should be challenged to prove their claim. A crosswalk comparing the CCSS and 2019 AL standards side by side should be produced.

The strategies based on place value so often referred to in the standards are often times great examples to show students how and why a standard algorithm works. They should not take the place of teaching the standard algorithms at the appropriate grades mentioned in the Standard Algorithm section. While these unspecified strategies may lend themselves to procedural/computational fluency, they tend to not be as efficient as the standard algorithm. As laid out in these standards, these inefficient strategies will be taught and well practiced for 2+ years prior to the standard algorithms being taught and required. It takes additional classroom time to help students make That shift when they should have started off learning and using the standard algorithms.

While it might seem novel to emphasize conceptual understanding and delay the requirement for the standard algorithms, there appears to be no empirical evidence that it will develop fluency or foster conceptual understanding. The development and delay of the standard algorithms for the basic math operations of addition, subtraction, multiplication, and division contribute to an inadequate preparation for algebra.

The 2019 AL standards are loaded with the pedagogy of the CCSS. Pedagogy does not belong in standards or even in a standards document. If it is felt pedagogy is needed, it should be separate from the standards and in a document of their own. Stripping pedagogy out of standards is relatively easy if one just focuses on the math. Here is the example used in the Standard Algorithm section.

> Use strategies based on place value, properties of operations, and/or the relationship between multiplication and division to find whole-number quotients and remainders with one-digit divisors and up to four-digit dividends.
> a. Illustrate and explain quotients using equations, rectangular arrays, and/or area models.
>
> The standard is complete as:
> Find whole-number quotients and remainders with one-digit divisors and up to four-digit dividends.
> It is still a CCSS standard even though the pedagogy has been stripped out.

If these new standards are basically the same as the CCSS, what is there about them that will improve achievement? Close the achievement gaps? Since there is little to no difference between the standards AL has been using and these new standards, what reason is there to think the current trends in achievement (including gaps) will change?

ABOUT THE AUTHOR: J.R. Wilson has 30 plus years' experience working in public education as an elementary classroom teacher, middle and high school math teacher, state department of education curriculum consultant, regional educational service agency staff development coordinator, and elementary principal. As a team member he has been involved in writing science and math standards. He has served on the Where's the Math? Executive Committee. He participated in the U.S. Coalition for World Class Math's review of the Common Core State Standards Initiative's College and Career Readiness Standards for Mathematics draft, coordinated the Coalition's review of the March 10, 2010 draft of the Common Core State Standards for Mathematics, and participated in the Coalition's review of the final set of math standards. See Common Does Not Equal Excellence.